Angle definitions

Plug-in Gait uses Cardan angles, modified in the case of the ankle angles, to represent both:

- Absolute rotations of the pelvis and foot segments and
- Relative rotations at the hip, knee, and ankle joints

These angles can be described either as a set of rotations carried out one after the other (ordered).

For more information about the use of Cardan angles to calculate joint kinematics, refer to Kadaba, Ramakrishnan and Wooten (1990) and Davis, Õunpuu, Tyburski and Gage (1991) (see Plug-in Gait references).

The rotations are measured about anatomical axes in order to simplify their interpretation.

For more information on joint angle descriptions, including the issues of gimbal lock and Codman's Paradox, see:

- Ordered rotations
- Angle goniometric description

Ordered rotations

To describe an angle using ordered rotations, the following are true:

- One element is 'fixed'. For absolute rotations the laboratory axes are fixed. The proximal segment axes are fixed for relative rotations.
- The second element 'moves'. This means the segment axes move for absolute rotations and distal segment moves for relative rotations.

A joint angle is then defined using the following ordered rotations:

- The first rotation (flexion) is made about the common flexion axis. The other two axes, abduction and rotation, are afterwards no longer aligned in the two elements.
- The second rotation (abduction) is made about the abduction axis of the moving element. The third rotation (rotation) is made about the rotation axis of the moving element.

Angle goniometric description

In addition to using ordered rotations, joint angles can also be described using goniometric information. Using goniometric definitions, a joint angle is described by the following:

- **Flexion** is about the flexion axis of the proximal (or absolute) element.
- **Rotation** is about the rotation axis of the distal element.
- **Abduction axis** 'floats' so as always to be at right angles to the other two.

Cardan angles work well unless a rotation approaching 90 degrees brings two axes into line. When this happens, one of the possible rotations is lost and becomes unmeasurable. Fortunately, this does not frequently occur in the joints of the lower limbs during normal or pathological gait. However this may occur in the upper limb and particularly at the shoulder. For more information, see Gimbal lock and also Codman's Paradox below.

Gimbal lock

Gimbal lock occurs when using Cardan (Euler) angles and any of the rotation angles becomes close to 90 degrees, for example, lifting the arm to point directly sideways or in front (shoulder abduction about an anterior axis or shoulder flexion about a lateral axis respectively). In either of these positions the other two axes of rotation become aligned with one another, making it impossible to distinguish them from one another, a singularity occurs and the solution to the calculation of angles becomes unobtainable.

For example, assume that the humerus is being rotated in relation to the thorax in the order Y,X,Z and that the rotation about the X-axis is 90 degrees.

In such a situation, rotation in the Y-axis is performed first and correctly. The X-axis rotation also occurs correctly BUT rotates the Z axis onto the Y axis. Thus, any rotation in the Y-axis can also be interpreted as a rotation about the Z-axis.

True gimbal lock is rare, arising only when two axes are close to perfectly aligned.

Codman's Paradox

The second issue however, is that in each non-singular case there are two possible angular solutions, giving rise to the phenomenon of "Codman's Paradox" in anatomy (Codman, E.A. (1934). The Shoulder. Rupture of the Supraspinatus Tendon and other Lesions in or about the Subacromial Bursa. Boston: Thomas Todd Company), where different combinations of numerical values of the three angles produce similar physical orientations of the segment. This is not actually a paradox, but a consequence of the non-commutative nature of three-dimensional rotations and can be mathematically explained through the properties of rotation matrices (Politti, J.C., Goroso, G., Valentinuzzi, M.E., & Bravo, O. (1998). Codman's Paradox of the Arm Rotations is Not a Paradox: Mathematical Validation. Medical Engineering & Physics, 20, 257-260).

Codman proposed that the completely elevated humerus could be shown to be in either extreme external rotation or in extreme internal rotation by lowering it either in the coronal or sagittal plane respectively, without allowing any rotation about the humeral longitudinal axis.

To demonstrate Codman's Paradox, complete the following steps:
1. Place the arm at the side, elbow flexed to 90 degrees and the forearm internally rotated across the stomach.
2. Elevate the arm 180 degrees in the sagittal plane.
3. Lower the arm 180 degrees to the side in the coronal plane.
   Observe that the forearm now points 180 degrees externally rotated from its original position with no rotation about the humeral longitudinal axis actually having occurred.
4. Note the difficulty in describing whether the fully elevated humerus was internally or externally rotated.

This ambiguity can cause switching between one solution and the other, resulting in sudden discontinuities. A combination of gimbal lock and Codman's Paradox can lead to unexpected results when joint modeling is carried out. In practice, the shoulder is the only joint commonly analyzed that has a sufficient range of motion about all rotation axes for these to be an issue. Generally, if you are aware of the reasons for the inconsistent data, you can manipulate any erroneous results by adding 180 or 360 degrees.

As Plug-in Gait uses Cardan (Euler) angles in all cases to calculate joint angles, they are subject to both Gimbal Lock in certain poses, and the inconsistencies that occur as a result of Codman's Paradox.

Plug-in Gait includes some steps to minimize the above effects by trying to keep the shoulder angles in consistent and understandable quadrants. This is not a complete solution however, as the above issues are inherent when using Cardan (Euler) angles and clinical descriptions of motion.